

ENERGY MODULATION OF THE ELECTRONS BY THE LASER FIELD IN THE WIGGLER MAGNET: ANALYSIS AND EXPERIMENT

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Abstract

Energy modulation of the electron beam after the interaction with the laser field in the wiggler magnet can be calculated using interference of the laser field and the field of spontaneous emission in the far field region of wiggler radiation. Quite often this approach gives a deeper insight on the process than traditional calculations where the effect of the laser field on the electron energy is integrated along the electron trajectory in the wiggler. We demonstrate it by showing the agreement between the analytical model and the experiment involving wiggler scan measurements with large detuning from the FEL resonance producing more than one order of magnitude variations in the amplitude of the energy modulation. The high sensitivity was achieved using the THz radiation from a sub-mm dip in the electron density that energy modulated electrons leave behind while propagating along the storage ring lattice. All measurements were performed at the BESSY-II electron storage ring.

ENERGY MODULATION

The energy gain/loss obtained by the electron in the interaction with the laser field E_x polarized in the horizontal plane and co-propagating the planar wiggler magnet together with the electron in z direction can be found by solving the equation [1]:

$$\frac{d\gamma}{dz} = \frac{e}{mc^2} E_x \cdot \beta_x, \quad (1)$$

where γ is the relativistic factor, $\beta_x = v_x/c$, where v_x is the horizontal velocity of the electron and c is the speed of light, e, m are the electron charge and mass, and

$$E_x = E_0 / \sqrt{1 + (z/z_0)^2} \sin[k(z - ct) + \psi] e^{-\tau^2/4\sigma_\tau^2}, \quad (2)$$

where k is the wave number of the laser field, $z_0 = ka_0^2/2$ is the Rayleigh length, a_0 is the waist size which is assumed to be in the center of the wiggler, $\psi = \psi_0 - \tan^{-1}(z/z_0)$, where ψ_0 is the phase of the wave at the beginning of the interaction with the electron at the entrance of the wiggler and σ_τ is the rms width of the laser pulse intensity.

For electron motion inside the wiggler one obtains:

$$\beta_x = -\frac{K}{\gamma} \sin(k_w z), \quad (3)$$

$$\beta_z = \sqrt{1 - \frac{1}{\gamma^2} - \beta_x^2} \approx 1 - \frac{1}{2\gamma^2} \left(1 + \frac{K^2}{2}\right) + \frac{K^2}{4\gamma^2} \cos(2k_w z)$$

where $k_w = 2\pi/\lambda_w$ and λ_w is the wiggler period,

$K = eB_0/k_w mc$, B_0 is the peak magnetic field and β_z is the normalized velocity along the wiggler, *i.e.*:

$$z(t) = c \int_0^t \beta_z(t') dt' \approx ct - \frac{1}{2\gamma^2} \left(1 + \frac{K^2}{2}\right) ct + \frac{K^2}{8k_w \gamma^2} \sin(2k_w z) \quad (4)$$

In what follows we consider on-axis electrons only. This assumption can be extended to all electrons in the electron bunch if the electron beam size $\sigma_{x,y} \ll a_0$.

Then, using (1) and (3) we write:

$$\frac{d\gamma}{dz} = \frac{eE_0 K}{2mc^2 \gamma \sqrt{1 + (z/z_0)^2}} \left\{ \cos(k(z - ct) + k_w z + \psi) - \cos(k(z - ct) - k_w z + \psi) \right\} e^{-\tau^2/4\sigma_\tau^2} \quad (5)$$

It is further convenient to define the resonance electron energy, also called an FEL resonance energy,

$$\gamma_r^2 = \frac{k}{2k_w} \left(1 + \frac{K^2}{2}\right) \text{ and assume a small energy spread}$$

$\Delta\gamma/\gamma_r$ such as to obtain:

$$k(z - ct) = -k_w z \frac{\gamma_r^2}{\gamma^2} - \frac{\xi}{2} \sin(2k_w z) \quad (6)$$

with $\xi = K^2/(2 + K^2)$. Using a generation function for Bessel functions [2] we found:

$$\begin{aligned} & \cos\left[k_w z \left(1 - (\gamma_r/\gamma)^2\right) - \xi/2 \sin(2k_w z) + \psi\right] - \\ & \cos\left[k_w z \left(1 + (\gamma_r/\gamma)^2\right) + \xi/2 \sin(2k_w z) + \psi\right] \approx \\ & [J_0(\xi/2) - J_1(\xi/2)] \cos\left[k_w z \left(1 - (\gamma_r/\gamma)^2\right) + \psi\right] \end{aligned}$$

where in the last step we retain only slowly varying terms with $1 - (\gamma_r/\gamma)^2 \ll 1$. Finally, using dimensionless variables in Eq. (6) [3]: $\hat{z} = z/L_w$, $\nu = N2\delta\gamma/\gamma_r$, and $q = L_w/z_0$, where L_w is the length of the wiggler with N periods, $\hat{\sigma}_\tau = \sigma_\tau/\tau_0$ and $\tau_0 = 2\pi N/kc$, one obtains:

$$\frac{d\gamma}{d\hat{z}} = \frac{eE_0 K L_w}{2mc^2 \gamma} \{JJ\} \frac{\cos(2\pi\nu\hat{z} - \tan^{-1}(q\hat{z}) + \psi_0)}{\sqrt{1 + (q\hat{z})^2}} e^{-\hat{z}^2/4\hat{\sigma}_\tau^2} \quad (7)$$

where definition $\{JJ\} = J_0(\xi/2) - J_1(\xi/2)$ is used following [4]. Introducing the laser pulse energy $A_L = (E_0^2/8\pi)(\pi a_0^2/2)\sqrt{2\pi}\sigma_\tau c$, Eq.(7) can be written as:

$$\begin{aligned} \left\langle \frac{d\gamma}{d\hat{z}} \right\rangle &= \frac{2}{mc^2} \sqrt{A_L \alpha \hbar \omega_{0s} \frac{K^2}{2 + K^2}} \{JJ\} \\ &\times \sqrt{\frac{2q}{(2\pi)^{1/2} \hat{\sigma}_\tau}} \frac{\cos(2\pi\nu\hat{z} - \tan^{-1}(q\hat{z}) + \psi_0)}{\sqrt{1 + (q\hat{z})^2}} e^{-\hat{z}^2/4\hat{\sigma}_\tau^2} \end{aligned} \quad (8)$$

where α is the fine structure constant, ω_{0s} is central frequencies of the field of spontaneous emission, and $\langle \dots \rangle$

defines averaging over one wiggler period. By integrating Eq.(8) one obtains for the amplitude of the energy modulation:

$$\Delta\gamma(q, \nu, \bar{\sigma}_\tau) = \frac{2}{mc^2} \sqrt{A_L \alpha \hbar \omega_{0s}} \frac{K^2}{2 + K^2} \{JJ\} f(q, \nu, \bar{\sigma}_\tau) \quad (9)$$

where:

$$f(q, \nu, \bar{\sigma}_\tau) = \sqrt{\frac{2q}{(2\pi)^{1/2} \bar{\sigma}_\tau}} \int_{-0.5}^{0.5} \frac{\cos(2\pi\nu\bar{z} - \tan^{-1}(q\bar{z}))}{\sqrt{1 + (q\bar{z})^2}} e^{-\bar{z}^2/4\bar{\sigma}_\tau^2} d\bar{z}$$

Figure 1 shows plots of $f(q, \nu, \bar{\sigma}_\tau)$ for various q and ν and $\bar{\sigma}_\tau$. Using the maximum value of $f(q=8, \nu=-0.7, \bar{\sigma}_\tau=0.25)=2.2$, one finds:

$$\Delta\gamma_{\max} \cong \frac{2}{mc^2} \sqrt{5A_L \alpha \hbar \omega_{0s}} \frac{K^2}{2 + K^2} \{JJ\} \quad (10)$$

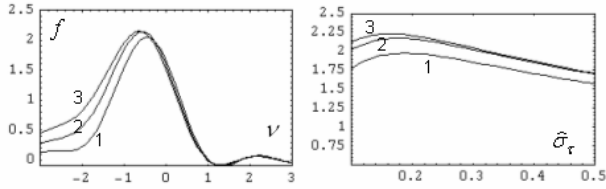


Figure 1. Function $f(q, \nu, \bar{\sigma}_\tau)$ for $\bar{\sigma}_\tau=0.25$ (left plot) and for $q=4, 6$, and 8 (curves 1, 2, and 3).

Alternatively, the energy gain/loss obtained by the electron in the interaction with the laser field can be calculated considering the interference of the field of its spontaneous emission in the wiggler and the laser field. This technique takes its roots in a so-called acceleration theorem [5,6] declaring that the very existence of spontaneous emission is mandatory if the acceleration by the external field is employed in a linear order to this field. Following this idea and using Parseval's theorem, we write for the amplitude of the energy modulation:

$$mc^2 \Delta\gamma_{\max} = 2 \frac{\pi a_0^2 c}{16\pi} \int E_s(t) E_L^*(t) dt = \quad (11)$$

$$2 \frac{\pi a_0^2 c}{16\pi} \int E_s(\omega) E_L^*(\omega) d\omega$$

where $E_s(\omega) = \sqrt{2/\pi} E_{0s} \sin((\omega - \omega_{0s})\tau_0/2)/(\omega - \omega_{0s})$ is Fourier component of the field of spontaneous emission, $E_L(\omega) = E_{0L} \sqrt{2\sigma_\tau} \cdot e^{-(\omega - \omega_{0L})^2 \sigma_\tau^2}$ is a Fourier component of the laser field, and ω_{0L} is the central frequency of the laser field. Further defining the energy of spontaneous emission (see, also [3]):

$$A_s = (E_s^2/8\pi) (\pi a_0^2/2) \tau_0 c = 5\alpha \hbar \omega_{0s} \frac{K^2}{2 + K^2} \{JJ\}^2 \quad (12)$$

radiating in the mode with a rms divergence of the intensity $\sigma'_R = \sqrt{2\lambda_s/\pi L_w}$, where $\lambda_s = 2\pi c/\omega_{0s}$, one can obtain from (11):

$$\Delta\gamma_{\max} \cong \frac{2}{mc^2} \sqrt{A_L A_s} \left\{ \sqrt{\sigma_\tau} \int e^{-4(x-x_{0L})^2 \sigma_\tau^2} \frac{\sin(x-x_{0s})}{(x-x_{0s})} dx \right\} \quad (13)$$

where we used substitution $x = \omega\tau_0/2$ and $x_{0s} = \omega_{0s}\tau_0/2$ and $x_{0L} = \omega_{0L}\tau_0/2$. The entire expression inside the figure bracket shows the overlapping of the spectra of the laser signal and the spectra of spontaneous emission signal. This expression is normalized in such a way that it is approximately one when $\omega_{0s} = \omega_{0L}$ and $\bar{\sigma}_\tau = 0.25$ corresponding to the optimal condition of the maximum energy modulation found previously. Finally we note that in the case of a large detuning from the FEL resonance a contribution of the second term with $\sin(x+x_{0s})/(x+x_{0s})$ could be comparable to a contribution of the main term with $\sin(x-x_{0s})/(x-x_{0s})$ and, thus, should be added to the overlapping integral.

EXPERIMENT

The energy modulation of electrons by the laser field was measured as a function of the detuning of the central frequency of the electron spontaneous emission in the wiggler from the central frequency of the laser field. The experiment was conducted at BESSY-II synchrotron light source [7]. The laser operated at a fixed wavelength of 800 nm while wiggler detuning covered large range of frequencies with the wavelengths changing from 200 nm to 1000 nm, *i.e.* in the range greatly exceeding the bandwidth of the laser pulse. As an illustration, Figure 2 shows normalized spectra of the laser field and the field of the electron spontaneous emission in the wiggler for a single set point during this scan.

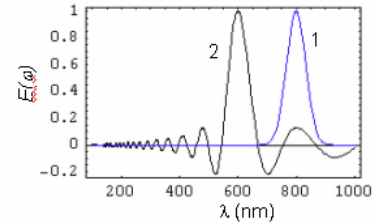


Figure 2. Spectra of the laser field (curve 1) and the field of spontaneous emission of electrons in the wiggler (curve 2).

Measurements of the energy modulation were performed indirectly using coherent THz radiation from a bending magnet 11 m downstream of the wiggler magnet. THz radiation was produced by the dip in the electron density distribution that energy modulated electrons leave behind after propagating the storage ring lattice with non-zero time-of-flight properties, a phenomena which is described elsewhere [8]. Use of the THz signal allowed us to obtain sufficient signal-to-noise ratio even with wiggler settings leading to very small amplitudes of the energy modulation. A typical example of THz spectra and the dip in the electron density distribution is shown in Figure 3.

The width and the magnitude of the dip are defined by the amplitude of the energy modulation of electrons and by time-of-flight parameters R_{51} , R_{52} , R_{56} of the lattice

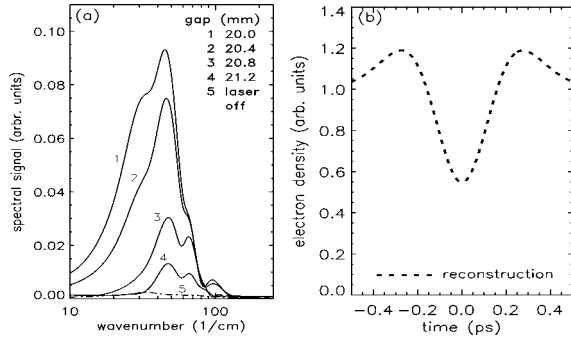


Figure 3. a) THz spectra for different wiggler gaps corresponding to different detunings near the FEL resonance, b) a dip in electron density determined from spectra measured at the FEL resonance and corrected for beamline transmission.

between the wiggler magnet and a source of the THz radiation. Figure 4 shows the magnitude of the dip calculated as a function of the amplitude of the energy modulation normalized on the relative energy spread of electrons σ_E . The following parameters were used:

$\sigma_E = 10^{-3}$, $R_{56} = -0.011$ m, $R_{51} = 5.5 \cdot 10^{-5}$, $R_{52} = 0.53$ m [9], the electron beam size and angle in the wiggler magnet $\sigma_x = 320$ μ m and $\sigma_x' = 55$ μ rad.

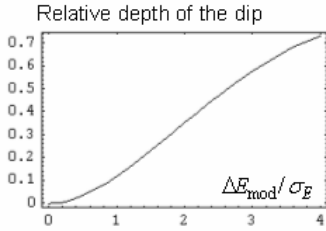


Figure 4. A relative magnitude of the dip as a function of the amplitude of the energy modulation ΔE_{mod} . The magnitude of one corresponds to 100% electron density modulation.

Variations in width and magnitude of the dip affect the THz signal, which was measured using an InSb-bolometer during the wiggler scan and plotted in Figure 5. Analyzing this measurement, we assumed that the THz signal is proportional to the square of the magnitude of the dip and that the magnitude of the dip itself is proportional to the amplitude of the energy modulation of electrons. We calculated the amplitude of the energy modulation using Eq. (13), which was, essentially, a calculation of the function $F(x_{0s}, x_{0L}, \sigma_\tau)$, i.e. the overlapping area between two spectra shown in Figure 2. These calculations are more accurate than calculations using Eq. (9) because of the assumption of a small detuning used there does not work for a broad scan of the wiggler wavelength as used in the experiment.

The width of the dip defines the spectra of the emitted THz signal [8] and, therefore, indirectly impacts the measurement because of the spectral dependence of the detector and THz beamline transmission. We accounted for this effect using empirically defined coefficient 0.75 for calculated intensity of THz radiation when the

amplitude of the energy modulation dropped below $\Delta E/\sigma_e < 3$. The other parameters used to obtain the fit were $N = 9.75$ (i.e. effective number of wiggler periods instead of 10 real periods), $\sigma_\tau = 45$ fs and a floor level of 10^{-4} (defined in the units used in Figure 5) as given by incoherent synchrotron radiation from the regular bunch.

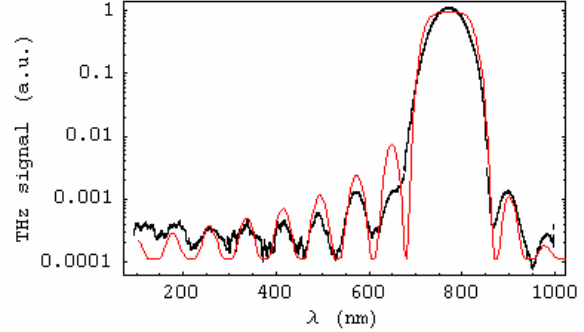


Figure 5. THz signal produced by the electron bunch with a dip in the electron density. The black curve shows the experimental result and the red curve the analytical fit.

CONCLUSION

Using a concept of the far field region we demonstrated that the energy modulation of electrons in the wiggler magnet by the laser light can be found by calculating the interference of the laser field and the field of the electron spontaneous emission in the far field region of the electron radiation in the wiggler. This allowed us to obtain a correct explanation for the measurements, where the wiggler detuning from the laser frequency covered a large range of frequencies exceeding the bandwidth of the laser field by many times.

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